Oddities in Asymmetric Coevolutionary Opinion Dynamics

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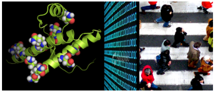

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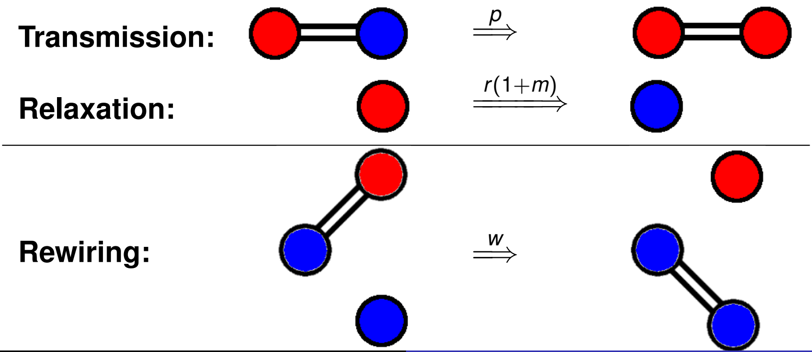
Cyclic Coevolutionary Model

S ⇒ I ⇒ S + topological coevolution

x: fraction of I-nodes; $m = \{1 - x\} - x$ network magnetization



Our Model	Adaptive SIS	Coevolutionary Voter Model
$SI \xrightarrow{p} II$ $I \xrightarrow{r(1+m)} S$	$SI \xrightarrow{p} II$ $I \xrightarrow{r} S$	$SI \xrightarrow{p/2} II$ $SI \xrightarrow{p/2} SS$
$SI + S \xrightarrow{w} SS + I$	$SI + S \xrightarrow{w} SS + I$	$SI + S \xrightarrow{w/2} SS + I$ and $SI + I \xrightarrow{w/2} II + S$

reparametrization: $w = \omega$, $p = (1-\omega)\rho$, $r = (1-\omega)(1-\rho)$

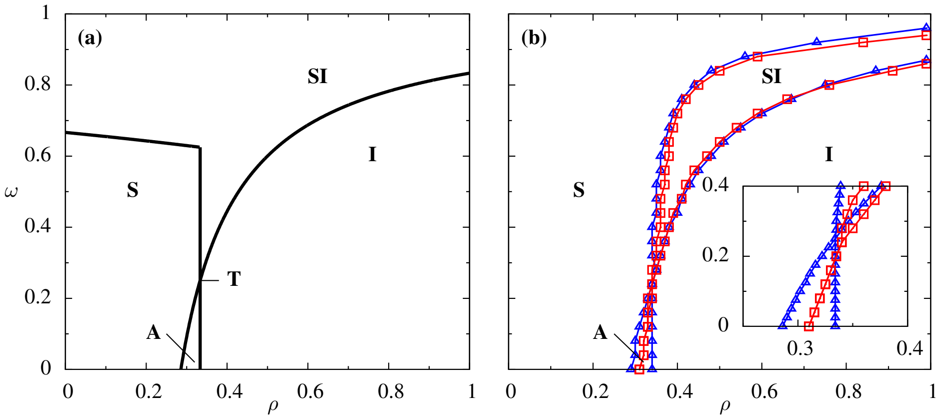
$$\frac{dx}{dt} = (1-\omega)\left(\rho z - 2\left(1-\rho\right)\left(1-x\right)x\right)$$

$$\frac{dy}{dt} = (1-\omega)\left(\rho z\left(\kappa\frac{z}{1-x}+1\right) - 4\left(1-\rho\right)\left(1-x\right)y\right)$$

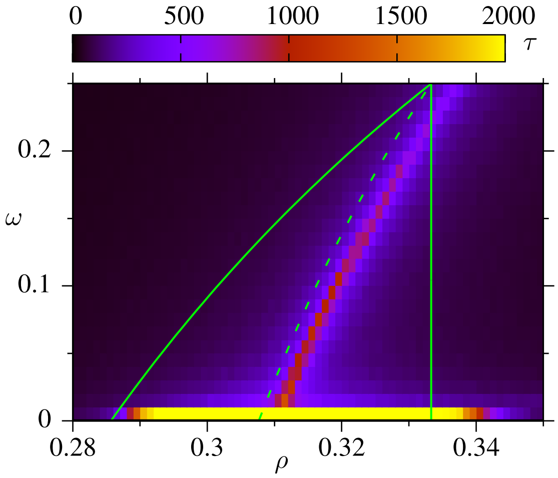
$$\frac{dz}{dt} = -z\left(\omega + (1-\omega)\left(\rho + 2\left(1-\rho\right)\left(1-x\right)\right)\right) - (1-\omega)\,\rho\kappa\frac{z^2}{1-x} + 4\left(1-\omega\right)\left(1-\rho\right)\left(1-x\right)y + 2\left(1-\omega\right)\rho\kappa\frac{\left(\langle k\rangle - y - z\right)z}{1-x}$$

Moment Closure

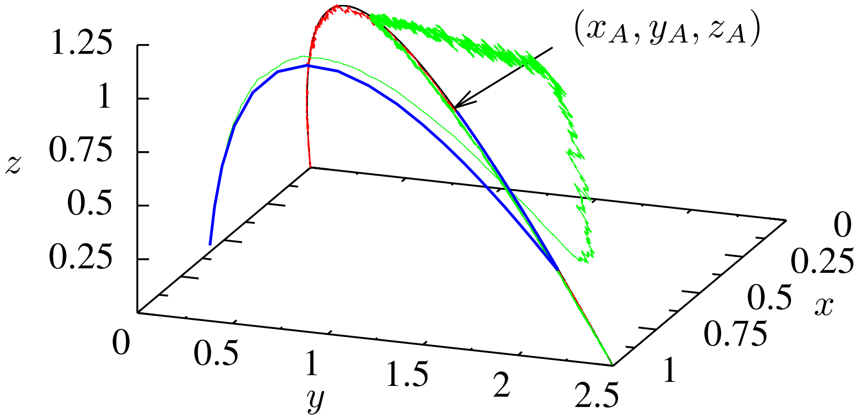
- κ encapsulates variance of underlying network's DD
- $\kappa = 1$ for ER graphs, $\kappa = \frac{\langle k\rangle - 1}{\langle k\rangle}$ for RR graph
- $\langle k\rangle$ is mean degree **and constant**



a) Phase diagram b) Change of asymptotic behavior in the PA with initial conditions (0.1, 0.025, 0.45), (0.5, 0.625, 1.25) and (0.9, 1.025, 0.45) (numerical integration of PA, blue triangles) and initially connected ER graphs with fractions 0.1, 0.5 and 0.9 of randomly assigned I-states (MC simulations, red squares). Mean degree $\langle k \rangle = 5$, MC simulations with $N = 5000$ nodes and results averaged over 100 realizations.



Color-coded convergence times τ in MC simulations within DE phase boundaries obtained from PA (solid green lines); maximum τ are expected at $x_A = 0.5$ (dashed green line). MC simulations with $N = 5000$, averaged over 100 runs. Mean degree $\langle k \rangle = 5$



Time evolution of state variables x, y and z along M_A (black line) for $\omega = 0.05$ and $\rho = 0.32$. MC trajectories from initial conditions $(0.01, 0.00025, 0.0495)$ (initial ER graph, red line) and $(0.8, 0.2, 0.2)$ (maximally random graph with respect to initial conditions, green line) end up in I-consensus, numerical integration from $(0.8, 0.2, 0.2)$ (blue line) in DE (x_A, y_A, z_A). Network size $N = 10^4$ in MC run.

Approximate slow manifold

$$M_A \begin{pmatrix} x \\ y_A\{x\} \\ z_A\{x\} \end{pmatrix} = \begin{pmatrix} x \\ x\frac{2x(x-\langle k\rangle)-2+\omega(1+(3+2\langle k\rangle-4x)x)}{2(\omega+\omega x-2)} \\ 2(1-x)x\frac{\langle k\rangle(\omega-1)+\omega+x-2\omega x}{\omega+\omega x-2} \end{pmatrix},$$

exact at triple point T.

With M_A and PA dynamics at hand, can we characterize metastability in the full system?

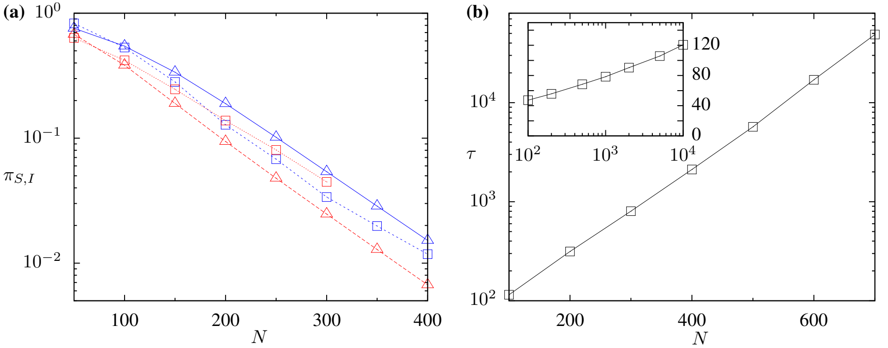
For $\omega = 0$ and $Z_A\{X\} = Nz_A\{X/N\}$, Master equation for RW in $X = xN$ along M_A

$$\frac{\partial[X]}{\partial t} = \rho Z_A\{X-1\}[X-1] + 2(1-\rho)(N-(X+1))\frac{X+1}{N}[X+1] - \rho Z_A\{X\} - 2(1-\rho)(N-X)\frac{X}{N}[X]$$

yields splitting probability

$$\pi_I\{X_0, N\} = \left(1 + \frac{\sum_{\mu=X_0}^{N-1}\prod_{X=1}^{\mu}\frac{2(1-\rho)(1-X/N)X}{\rho Z_A\{X\}}}{1+\sum_{\mu=1}^{X_0-1}\prod_{X=1}^{\mu}\frac{2(1-\rho)(1-X/N)X}{\rho Z_A\{X\}}}\right)^{-1}$$

$\pi_I\{X_0, N\} = X_0/N$ on neutrally stable manifold.



a) System-size dependent splitting probabilities for $\omega = 0$ computed analytically (triangles) and taken from MC simulations (squares). π_I computed for $\rho = 0.305$ (starting from $x_0 = 0.9$, blue symbols), π_S computed for $\rho = 0.315$ (starting from $x_0 = 0.1$, red symbols). b) Convergence times in MC simulations as a function of system size for $\omega = 0$ and $\rho = 0.3$. Inset $\omega = 0.05$.

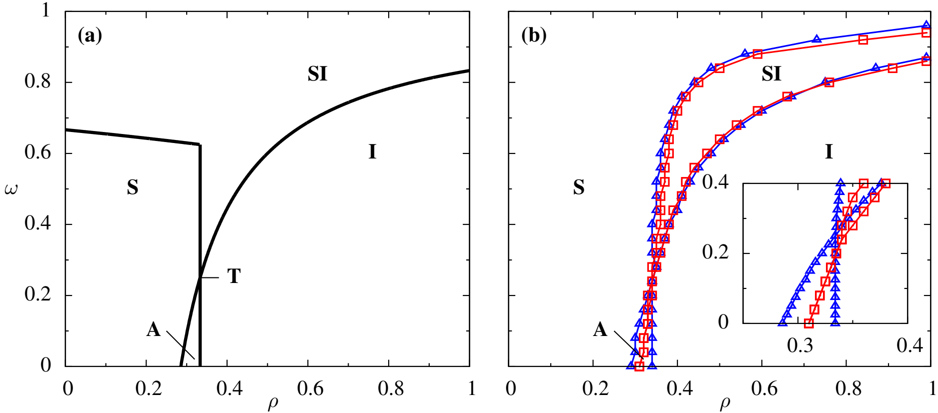
collapsing system to 1-dim RW along slow manifold gives good *qualitative* understanding of metastable regime:

- scaling of π_I with N
- predetermination of consensus states for $N \to \infty$
- parameter region of maximum convergence times

However it delivers dissatisfying *quantitative* description of full system:

- plugging in exact SM sampled from MC simulations brings no improvement
- reason: transversal fluctuations in conjunction with presence of stable DE
- dilemma: stochastic framework applicable only when $N \to \infty$

Symmetric Coevolutionary VM	Asymmetric Coevolutionary VM
one ODE	3-dim system of ODEs
magnetization conserved	magnetization not conserved
fragmented phase	no fragmentation
neutrally stable SM in active phase	SM generally not neutrally stable
$\pi_I\{X_0, N\} = X_0/N$	$\pi_I\{X_0, N\} \to 0, 1$
stochastic consensus	stochastic, dynamical consensus
stochastic bistability	dynamical bistability
robust with respect to κ	choice of κ crucial



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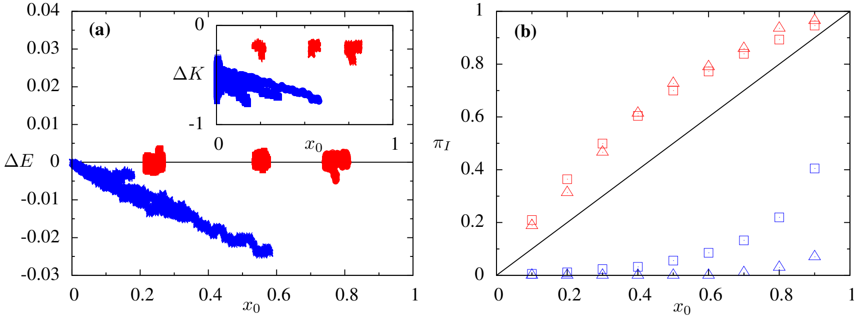
At triple point $\omega_T = 2/(1+\langle k \rangle)$, $\rho_T = 2/(3+\langle k \rangle)$

symmetric VM emulated for nontrivial parameter combination

- SM neutrally stable
- $\langle k_S \rangle = \langle k_I \rangle$
- flipping all node spins in DE yields another DE

Moreover: equipartition of transmission, relaxation and rewiring events.



a) Balance of events $\Delta E = \rho z - 2(1-\rho)(1-x)x$ and (inset) of mean degrees $\Delta K = \langle k_S \rangle - \langle k_I \rangle$ for bursts of simulations from $x_0 = 0.2, 0.5, 0.8$, recorded $N = 10^5$ and $10 \leq t \leq 100$. b) Splitting probabilities for $N = 100$ (squares) and $N = 1000$ (triangles). Simulations averaged over 10000 runs from initially connected ER graphs and with mean degree $\langle k \rangle = 5$.

Given certain ergodic properties of a network process in dynamic equilibrium, steady state averages imply the steady state of distributions they arise from. These steady-state distributions are solely determined by model parameters (S. Wieland, T. Aquino and A. Nunes, EPL 97, 18003, 2012).

Already for adaptive SIS model in active phase, steady-state topologies of S- and I-ensemble are identical for nontrivial choice of parameters (S. Wieland, A. Parisi and A. Nunes, EPJ-ST 212 (1), 99-113, 2012).

Can different microscopic mechanisms give rise to identical (equilibrium) ensemble behavior?

If so, are there ”canonical” microscopic dynamics that encompass a wide class of models for (nontrivial) parameter combinations?

Asymmetric coevolutionary opinion dynamics

- are highly sensitive to initial network topology.
- for $\kappa = 1$, yield dynamical consensus and bistability thereof, lack fragmentation.
- display metastability with unique features whose quantitative description demands for improved stochastic framework.
- reduce to steady-state adaptive SIS for $m = 0$ ($x = 0.5$)
- emulate steady-state symmetric VM at triple point of PA, partially also in the full system.
- question whether equilibrium ensemble statistics determine microscopic dynamics they emerge from.